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Formulation of the Optimal Strategy of Material Inventory Management in Industrial Enterprise (Probabilistic Model)

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Abstract

In the paper the design of the probabilistic model of a material inventory is discussed. The objective was the elaboration of the optimal strategy of the material inventory management in manufacturing. The author discusses probabilistic model of inventory aiming at the elaboration of the best management of raw materials in industrial enterprise. The model feasibility has been checked based on numerical data and concerning various products of manufacturing. Proposed inventory probabilistic model allows the formulation of an optimal strategy of the (R, Z) type of the raw materials management with profit consideration coming from low cost components in working.

1. Introduction

For purposes of ensuring an enterprise production continuity, the inventory of various production inputs are generated and maintained. Changing level of production costs and an impact of thge proper inventory management system upon costs do justify often extensive works on the elaboration of an optimal strategy for effectively handling inventoiry systems. Those call usually for mathematical modelling based on probabilistic nature of the inventory.
The level of raw material in stock of an industrial enterprise differs over time, depending on the input and output of the production system. Input of materials depends on the supply amount and time interval between the deliveries, output depends on the level of production consumption (Fig. 1). Time between consecutive material deliveries (delivery cycle) as well as its production consumption is assumed to be stable and controlled.

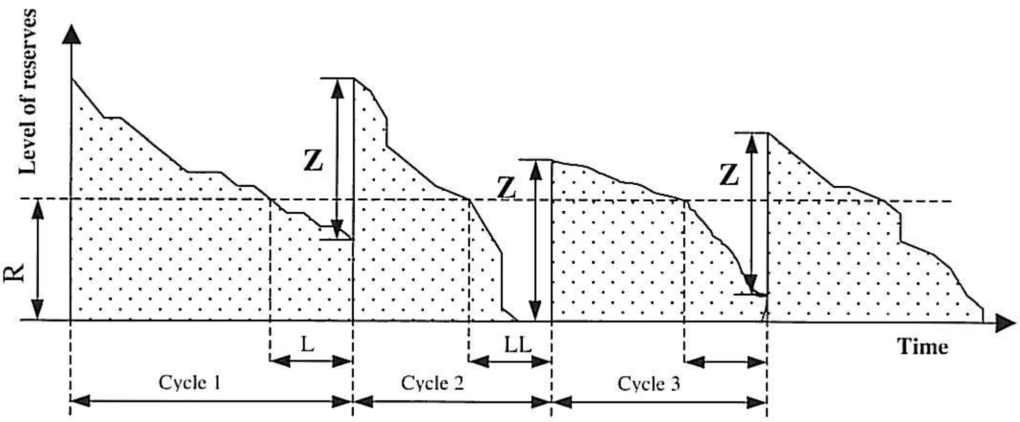

Figure 1. Change in level of a material reserves in magazine factory: Z – value of order of special kind of material, R – value of safety reserve, L – period of delivery.

Source: author's elaboration.

Many times inventory level and changes show some opposite tendencies affecting its overall behavior. High inventory level of the production inputs is associated with high costs of storage; similar is the case of low inventory level not ensuing production continuity not. Thus, the inventory level is to be maintained at a certain, determined by properties of the production system value. Any discrepancy between real inventory level and that required for the production system causes an extra costs to be incurred. Therefore, cost minimization should be main goal of the inventory management, while the main concern for optimization should be the equilibrium among costs of buying, storing, and requirements of the production flows. [1,6].

2. Construction of the probabilistic model

In order to construct a probabilistic model of inventory management certain symbols are introduced:

D – mean production consumption for special kind of material in certain time (e.g. medium years consumption),
Z – value of order of special kind of material,
D/Z – mean number of orders of special kind of material in certain time (for ex. during one year),
R – value of safety inventory (for special kind of material),
K – constant costs of orders,
h – single cost of storing,
p – single cost in the case of lack of the inventory in stock,
L – period (time) of delivery (time for order realisation),
v – production consumption for special kind of material in period of delivery,

E(v) – desired value of production consumption for special kind of material in period of delivery,
g(v) – probability distribution of production consumption of material delivery period (random variable density probability function),
b – mean level of lacking stores in period of delivery,
B – mean level of lacking stores for special material in certain investigation time,
E(B) – expected value of mean level of lacking stores for special material in certain time,
A – uniform distribution – upper limit of function,
E – operator of expected value,
F(Z,R) – main function of model with decisive variables Z and R.

It should be noted that by the end of delivery cycle expected level of inventory of a material in the store is R–E(v), but immediately after the arrival of the order (on the beginning of cycle) it is Z+R–E(v). Expected medium level of inventory for the material in the cycle, when $v \le R$ is calculated:

$$\frac{(Z+R-E(v))+(R-E(v))}{2}=\left(\frac{Z}{2}+R-E(v)\right) \tag{1}$$

In the case when $v > R$ is expected in the cycle (on the period of delivery), medium level of lacking inventory is calculated:

$$b=\int_R^{\infty}(v-R)\,g(v)\,dv \tag{2}$$

Expected in certain time (e.g. during one year) medium level of lacking stocks of the material (B) is calculated:

$$E(B)=b\cdot\frac{D}{Z} \tag{3}$$

Having calculated certain levels of material inventory, we can calculate appropriate costs, multiplicating appropriate single costs h and p with expressions (1) and (3).
Thus, main function of optimisation in the probabilistic model of the inventory management at a certain time can be expressed:

$$E[F(Z,R)]=K\frac{D}{Z}+h\left[\frac{Z}{2}+R-E(v)\right]+p\frac{D}{Z}b\rightarrow \min \tag{4}$$

First component of the equation represents mean cost of constants, the second is the mean cost of storing the reserves, while the third component – mean cost in the case of the lack of inventory. Changes of quantities for (Z and R) in the main equation will be minimal when Z and R are optimal [3,4]. For optimal values of the pair (R,Z) it is necessary that first partial derivatives of main equation (1) in comparison of Z and R values will be zero.

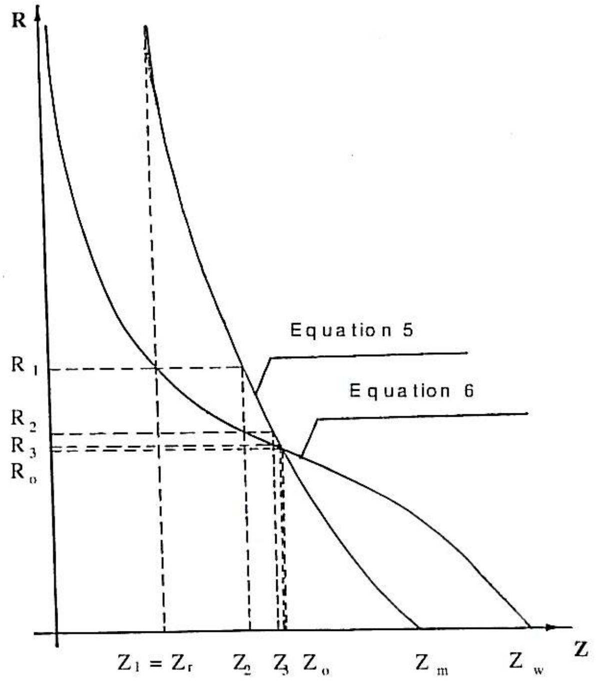

Figure 2. Scheme of iteration researches Z0 and R0

Making appropriate calculations, optimal values of order and store level R expressions can be found:

$$Z_0 = \sqrt{\frac{2D(K + pb)}{h}} \tag{5}$$

$$\int_{Ro}^{\infty} g(v)dv = \frac{hZ_0}{pD} \tag{6}$$

It should be emphasized that expressions (5) and (6) can not be used for direct calculation of optimal values of (Z_0, R_0) – (Fig. 2.). For that reason an iterative method of seeking Z_0 and R_0 values in the finite number of steps was elaborated [2]. The method implies that the inequality included in the equation 7 must be satisfied:

$$\frac{pD}{h} > \sqrt{\frac{2D[K + pE(v)]}{h}} \tag{7}$$

i. e. $Z_w > Z_m$ (Fig. 2)

3. Algorithm of setting the optimal values z_0 and r_0

Beginning the iterative process from the first probable meaning of Z value equal, with the increase of iteration numbers the value of Z_i increases, when R_i value decreases. So iterative process is quickly convergent. Because of great difficulties of calculating by hand the R_0 and Z_0 values, it is recommended to use computer techniques for these calculations. For this purpose an operative scheme of R_0 and Z_0 calculations was constructed as a computer program. This program performs calculations until the difference $R_{i+1}-R_i$ values is adequately low (e.g. 0.00001). It means, that two calculated values are almost equal. For the optimal value R_0 we use then R_{i+1} value, because $R_0 \cong R_{i+1}$. Optimal value of Z_0 we estimate on the basis of $R_0(R_{i+1})$ (Fig. 3, Fig. 4).
It should also be noted, that in the case when there is even distribution of probability of production consumption of materials, the expressions (5) and (6) are to be solved directly, i.e. optimal values of R_0 and Z_0 could be presented as expressions:

$$R_0 = A\left[1 - p^{-1}\sqrt{\frac{2Kh}{D - Ah}}\right] \tag{8}$$

$$Z_0 = D\sqrt{\frac{2K}{h(D - Ah)}} \tag{9}$$

Deduction of formulas (8), (9) is based on the case of even probability of production consumption of materials:

$$g(v) = \begin{cases} 0, & if \quad v \notin]0, A[\\ \frac{1}{A}, & if \quad v \in]0, A[\end{cases} \tag{10}$$

The integral, which can be seen in the expression (6) can be presented by using elementary functions. Of course, in the total case such a simplification is not possible and iterative process of estimation R_0 and Z_0 should be then realized.

4. Empiric verification of the model (on real data)

The constructed probabilistic model is under empirical verification. The practical verification of the constructed model was performed for every kind of material used during serial manufacturing of furniture (kitchen sets, combined set, chairs, armchairs). In the calculations numerical data were used from a 6–years period of production activities of furniture factories. The verification showed, that proposed here iterative method of solving the model of reserves leads to estimation of optimal values of reserves R_0 and order Z_0 with minimal costs, and consequently leads to determination of optimal (R, Z) type strategy of material inventory management in the factory.

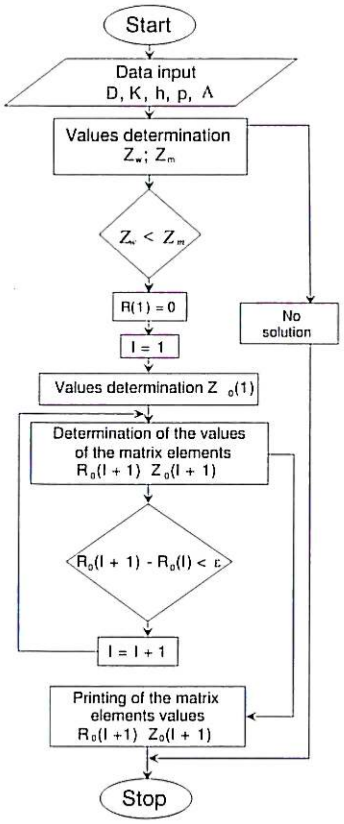

Figure 3. R_0 and Z_0 optimal values searching block diagram

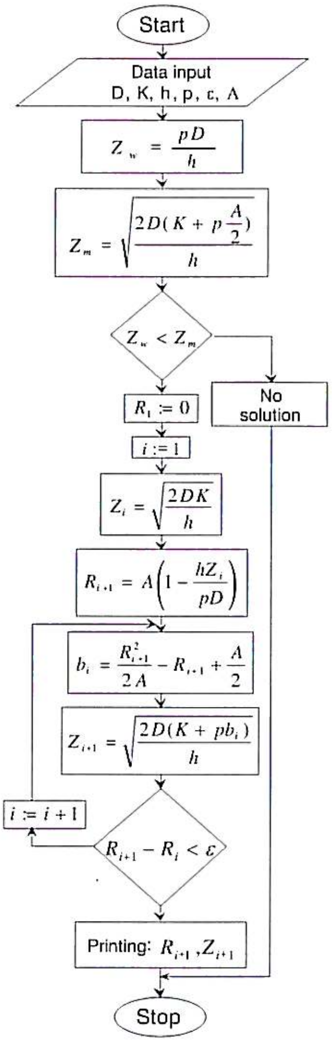

Figure 4. R_0 and Z_0 optimal values searching block diagram with uniform distribution

5. Optimal strategy setting

The main rule of optimal strategy of inventory management is: when the level of reserves of special based material in the store reach its value R_0, the order should be made equal to the value of Z_0 so that mean costs concerned with inventory in accepted time is minimal.

The constructed probabilistic model of wood inventory management has a methodological value because it shows a method of working out an optimal strategy of material inventory for serial and multi assortment production of furniture and probably other products in furniture industry.

6. Conclusions

The model has been verified at an attainable sample of industrrial enterprises using the numerical data concerning the different assortment of materials in the serial production of furniture. The proposed probabilistic model makes it possible to elaborate the optimal strategy of the type (R, Z) in the inventory management of materials of the manufacturing enterprise with large–lot production. The strategy is based on the minimal expenses connected with the supply of materials.

References

[1] Wagner H. M., *Principles of Operations Research with Applications to Managerial Decisions*, Panstwowe Wydawnictwo Ekonomiczne, Warszawa 1990.
[2] Żukowski P., *Management of an Industry Enterprise*, Panstwowe Wydawnictwo Naukowe, Warszawa – Wroclaw 1994.
[3] Żukowski P., *Concept of the Optimal Strategy of Material Reserves Management on the Base of Probabilistic Model in the Industrial Company*, in: "Manufacturing Engineering: 2000 and Beyond", Storrs: University of Connecticut, 1996, pp. 416 – 418.
[4] Żukowski P. and Prusak Z., *Design and Analysis of the Dynamic Management Model of a Manufacturing Company*, in: The 15th International Conference on Production Research: Manufacturing for a Global Market, Limerick: M.T. Hillery & H.J. Lewis, University of Limerick (Ireland,) 1999, pp. 807 – 810.
[5] Żukowski P., *The Basis of Management Sciences*, Rzeszow: Wydawnictwa Politechniki Rzeszowskiej, 2006
[6] Żukowski P., *Basis of Scientific Work Organization with chosen Management Problems*, Opole: Wydawnictwa Akademii Administracji i Zarzadzania, 2008.